Universität Konstanz

Indeterministic Causal Models and Historical Conditionals

Antje Rumberg

Antje.Rumberg@uni-konstanz.de

Department of Philosophy
University of Konstanz

Workshop: Modeling Causal Dependencies in Formal Semantics

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WHATIF
WASWÄREWENN DFG Project: What if? Subproject P7 "Alternatives for the Future"

Overview

Historical Conditionals and Real Possibilities

Historical Conditionals

- the underlying notion of possibility is real possibility

Real Possibilities

- alternative possibilities for the future to evolve
  given the past course of events and the laws of nature

Examples

(a) If Bill flips the switch, the light goes on.
(b) If Ann does not miss the bus, she will be on time.

(a') If Bill had flipped the switch, the light would have been on.
(b') If Ann had not missed the bus, she would have been on time.

Schulz on Causal Models and Conditionals

A model $\mathcal{M}$ for a propositional language $\mathcal{L}$

- a set W of possibilities $w : \mathrm{At} \to \{0, 1\}$;
- a causal dynamics $D = \langle B, F \rangle$;
  - $B \subseteq \mathrm{At}$ is the set of background variables;
  - $F : P \mapsto \langle Z_P, f_P \rangle$
    - $dom(F) = \mathrm{At} \setminus B$;
    - $Z_p \subseteq \mathrm{At}^n$ represents the variables that P immediately depends on;
    - $f_p : \{0, 1\}^n \to \{0, 1\}$ specifies the dependency of P on Z_p.

$F(C)$	A	C
	1	1
	0	0

The basis of a possibility w

- the smallest $w' \subseteq w$ whose causal closure $\mathrm{ccl}(w')$ is w
  - Closing w' causally:
    whenever $P \notin dom(w')$ and for all $X_i \in Z_p$, $X_i \in dom(w')$, we let $w'(P) = f_P(w'(X_1), \ldots, w'(X_n))$

The Prospects of Schulz's Theory

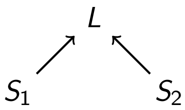
Conditionals

- $\mathcal{M}, w \vDash \phi \gg \psi$ iff for all $w' \in \mathit{Intervene}_{\mathcal{M}}(w, \phi)$, $\mathcal{M}, w' \vDash \psi$
  - $\mathit{Intervene}_{\mathcal{M}}(w, \phi)$
    $=$ the set of ϕ-possibilities w' s.t.
    (i) the overlap of the basis $b_{w'}$ with the basis b_w is maximal;
    (ii) the difference of the basis $b_{w'}$ with the basis b_w is minimal.

Example

Suppose there is a circuit such that the light is on exactly when both switches are in the same position (up or not up). At the moment switch one is down, switch two is up and the lamp is out.

- If switch one had been up, the lamp would have been on.



$F(L)$	S_1	S_2	L
	1	1	1
	1	0	0
	0	1	0
	0	0	0

w	w_1	w_2
$\neg L$	L	$\neg L$
S_2	S_2	$\neg S_2$
$\neg S_1$	S_1	S_1

Determinism

- interventions require miracles

Example

C
$\uparrow$
B
$\uparrow$
A

$F(B)$	A	B
	1	1
	0	0

$F(C)$	B	C
	1	1
	0	0

Suppose that $w(A) = 1, w(B) = 1$ and $w(C) = 1$.

- If B had not been the case, C would not have been the case.

w	w_1	w_2	w_3
C	$\neg C$	C	$\neg C$
B	$\neg B$	$\neg B$	$\neg B$
A	A	A	$\neg A$

Schulz on Causal Models and Conditionals

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C
$\uparrow$
A

$F(C)$	A	C
	1	1
	0	$*$

The basis of a possibility w

- the smallest $w' \subseteq w$ whose causal closure $\text{ccl}(w')$ is w
  - Closing w' causally:
    whenever $P \notin dom(w')$, for all $X_i \in Z_p$, $X_i \in dom(w')$ and $\langle w'(X_1), \ldots, w'(X_n) \rangle \in dom(f_P)$, we let $w'(P) = f_P(w'(X_1), \ldots, w'(X_n))$

The Prospects of Schulz's Theory

Example

Consider a man—call him Jones—who possessed the following disposition as regards wearing his hat. If the man on the news predicts bad weather, Mr Jones invariably wears his hat the next day. A weather forecast in favor of fine weather, on the other hand, affects him neither way: he puts his hat on or leaves it on the peg, completely at random. Suppose, moreover, that yesterday bad weather was prognosed, so Jones is wearing his hat.

- If the weather forecast had been in favor of fine weather, Jones would have been wearing his hat.

hat
↑
bad

F_{hat}	bad	hat
	1	1
	0	$*$

w	w_1	w_2
hat	hat	¬hat
bad	¬bad	¬bad

Non-Determinism

- creates gaps in the causal structure

Example (revised)

Consider a man—call him Jones—who possessed the following disposition as regards wearing a hat. If the man on the news predicts bad weather, Mr Jones invariably wears his blue hat the next day. If, on the other hand, the weather forecast is in favor of fine weather, he puts his blue hat on or his red one, completely at random. Suppose, moreover, that yesterday bad weather was prognosed, so Jones is wearing his blue hat.

- If the weather forecast had been in favor of fine weather, Jones would have been wearing his blue hat or his red one.

blue
↑
bad

F_{blue}	bad	blue
	1	1
	0	$*$

w	w_1	w_2	w_3
blue	blue	red	green
bad	¬bad	¬bad	¬bad

Schulz on Causal Models and Conditionals

A model $\mathcal{M}$ for a propositional language $\mathcal{L}$

- a set W of possibilities $w : \mathrm{At} \times T \rightarrow \{0,1\}$;
- a causal dynamics $D = \langle B, F \rangle$;
  - $B \subseteq \mathrm{At}$ is the set of background variables;
  - $F : P \mapsto \langle Z_P, f_P \rangle$
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$F(C)$	A	C
	1	1
	0	$*$

The basis of a possibility w

- the smallest $w' \subseteq w$ whose causal closure $\mathrm{ccl}(w')$ is w
  - Closing w' causally:
    whenever $\langle P, t_k \rangle \notin dom(w')$, for all $X_i \in Z_p$, $\langle X_i, t_{k-1} \rangle \in dom(w')$ and $\langle w'(X_1, t_{k-1}), \ldots, w'(X_n, t_{k-1}) \rangle \in dom(f_P)$, we let $w'(P, t_k) = f_P(w'(X_1, t_{k-1}), \ldots, w'(X_n, t_{k-1}))$

Schulz on Causal Models and Conditionals

A model $\mathcal{M}$ for a propositional language $\mathcal{L}$

- a set W of possibilities $w_t : \text{At} \times \{t' \mid t' \leq t\} \rightarrow \{0, 1\}$;
- a causal dynamics $D = \langle B, F \rangle$;
  - $B \subseteq \text{At}$ is the set of background variables;
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C
$\uparrow$
A

$F(C)$	A	C
	1	1
	0	u

The basis of a possibility w

- the smallest $w' \subseteq w$ whose causal closure $\text{ccl}(w')$ includes w
  - Closing w' causally:
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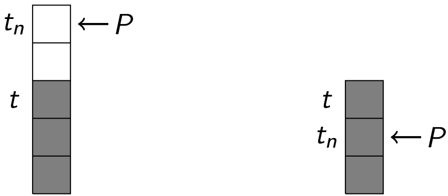
The Prospects of Schulz's Theory

Two kinds of interventions

Suppose we want to evaluate $P(t_n) \gg Q(t_{n+k})$ w.r.t. w_t.

$t_n > t$ $\qquad$ $t_n \leq t$



Conditionals

$\mathcal{M}, w \vDash \phi \gg \psi$ iff for all $w' \in Intervene_{\mathcal{M}}(w, \phi)$, $\mathcal{M}, \text{ccl}(w') \vDash \psi$

Schulz and Historical Conditionals?

Schulz's account

- Similarity in terms of bases

  ⇒ provides perspicuous account of causal closeness

- Deterministic causal laws

  ⇒ interventions require miracles

- Non-deterministic causal laws

  ⇒ create gaps in the causal structure

Historical conditionals

- Indeterministic causal laws

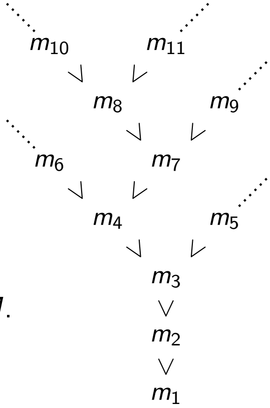
  ⇒ limited range of alternative lawful possibilities

Branching Time Structures

A *branching time structure* $\mathcal{M} = \langle M, < \rangle$ is a non-empty strict partial ordering (irreflexive, asymmetric, transitive) of moments such that

(i) there is no backward branching;

(ii) any two elements in M have a greatest common lower $<$-bound in M.



The Theory of Branching Time

Branching Time Structures

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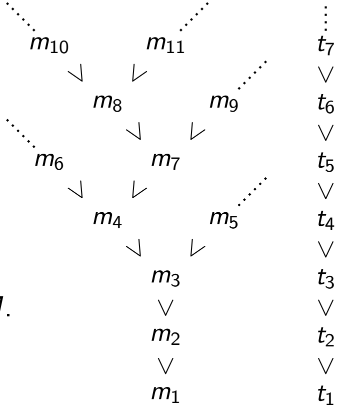

The Theory of Branching Time

Branching Time Structures

A *branching time structure* $\mathcal{M} = \langle M, < \rangle$ is a non-empty strict partial ordering (irreflexive, asymmetric, transitive) of moments such that

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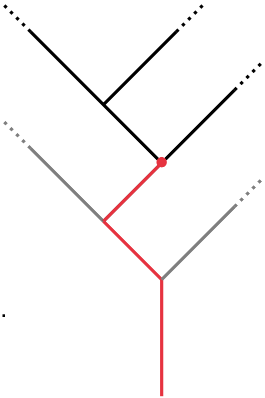
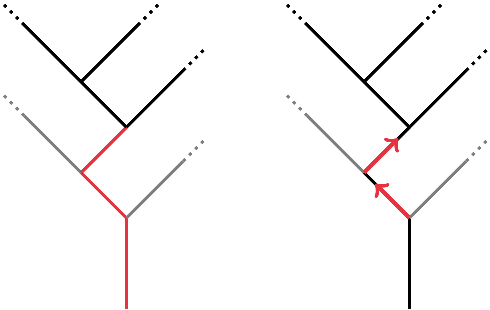


The Theory of Branching Time

Transition Semantics

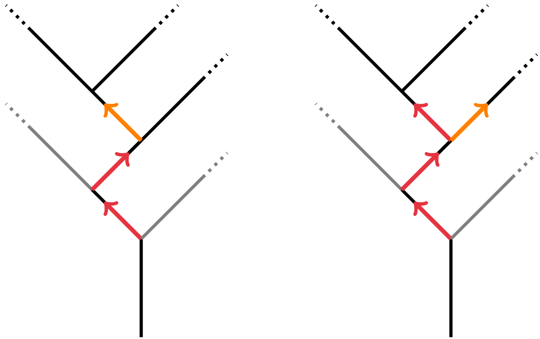
- possible courses of events are modeled by chains of transitions
- a transition is an <initial $\rightarrowtail$ outcome>-pair that specifies one immediate possible future continuation at a branching point



The Theory of Branching Time

Transition Semantics and Historical Conditionals

- Two kinds of intervention
  - extend the transition parameter
  - shift the transition parameter



Indeterministic Causal Models

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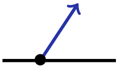
Potentialities and their manifestations

- Potentialities

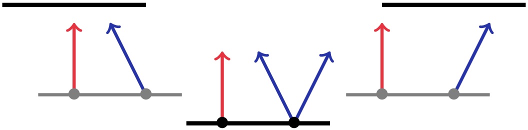
  = modal properties of objects in momentary circumstances
- Manifestations

  = transitions that locally point toward the future



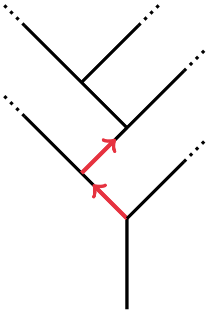
A causal dynamics in terms of potentialities

- every circumstance c determines a set of transition sets
- every possible combination over the transition sets in that set determines a possible future continuation of c



Branching Time Models for Real Possibility

- the causal closure of the initial circumstance is a branching time model for real possibility
- each structural transition is spanned by a non-empty set of transitions that capture the manifestations of potentialities
- the basis of a chain of structural transitions (i.e. of a possible course of events) is the chain of the corresponding sets of (indeterministic) manifestation-transitions



Indeterministic Causal Models

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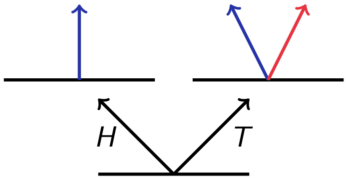
Example

Consider a man—call him Jones—who possessed the following disposition as regards wearing a hat. In the evening he tosses a coin. If the coin lands heads, Mr Jones invariably wears his blue hat the next day. If, on the other hand, the coin lands tails, he puts his blue hat on or his red one, completely at random. Suppose, moreover that yesterday the coin landed heads, so Jones is wearing his blue hat.

- If the coin had landed tails, Jones would have been wearing his blue hat or his red one.

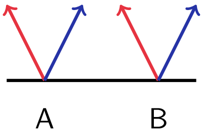


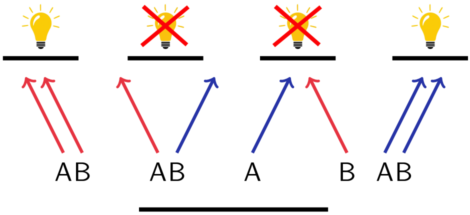
Indeterministic Causal Models

Example

Suppose there is a circuit such that the light is on exactly when both switches are in the same position (up or not up). Ann can flip switch one or refrain from doing so, and Bill can flip switch two or refrain from doing so. Initially both switches are down, and the lamp is out. Then Ann flips switch one, while Bill does nothing. Accordingly, the lamp is still out.

- If Bill had flipped switch two, the lamp would have been on.





Wrapping Up

Schulz's Account

- ontic conditionals
- possible worlds framework
- incomplete possible worlds
- (non-)determinism
- causal models based on propositional variables
- similarity in terms of bases

Present Proposal

- historical conditionals
- branching time framework
- transition sets
- indeterminism
- causal models based on potentialities
- similarity in terms of bases (?)